

# Poincaré invariance constraints on non-relativistic effective field theories

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We discuss Poincaré invariance in the context of non-relativistic effective field theories of QCD. We show, in the cases of the HQET and pNRQCD, that the algebra of the generators of the Poincaré transformations imposes precise constraints on the form of the Lagrangian. In the case of the HQET they are the relations formerly obtained by reparametrization invariance.

## 1. INTRODUCTION

The heavy quark effective field theory (HQET) is the effective field theory of QCD suitable for describing heavy-light mesons [1]. The heavy-quark four momentum can be split into a large and a small component:  $p = mv + k$ , where  $m$  is the heavy quark mass,  $v$  a unit vector and  $k$  a residual momentum of the order of  $\Lambda_{\text{QCD}}$ . The large and small component fields  $\psi_v^\pm$  are defined in terms of the heavy-quark field  $\Psi$  as  $\psi_v^\pm(x) = \frac{1 \pm \not{v}}{2} e^{imv \cdot x} \Psi(x)$ . The HQET is obtained by integrating out the small component field  $\psi_v^-$ . The HQET Lagrangian  $\mathcal{L}$  depends on  $v$  either explicitly or via the heavy-quark field  $\psi_v^+$ . This dependence is fictitious since  $v$  is just a parameter that arbitrarily defines the way one splits the heavy-quark momentum into a large and a small part. Therefore,  $\mathcal{L}$  must be a combination of reparametrization invariant operators, i.e. non-reparametrization invariant operators must appear in special combinations inside the HQET Lagrangian. As a consequence, the corresponding Wilson coefficients satisfy some exact relations valid at any order in perturbation theory [2].

In the following I will be concerned with the relation between reparametrization and Poincaré invariance. In the first part I will show how to derive the same relations obtained from reparametrization invariance, by imposing the Poincaré algebra on the generators of the

Poincaré transformations of the HQET, i.e. without introducing the parameter  $v$ . The method may be extended to non-relativistic effective field theories (EFTs) where the relation of the effective degrees of freedom with the original ones is less known than in the case of the HQET. As an application, in the second part I will derive the constraints imposed by Poincaré invariance on potential NRQCD (pNRQCD).

## 2. POINCARÉ INVARIANCE

For any Poincaré invariant theory the generators  $H, \mathbf{P}, \mathbf{J}, \mathbf{K}$  of time translations, space translations, rotations, and Lorentz transformations satisfy the Poincaré algebra:

$$[\mathbf{P}^i, \mathbf{P}^j] = 0, \quad (1)$$

$$[\mathbf{P}^i, H] = 0, \quad (2)$$

$$[\mathbf{J}^i, \mathbf{P}^j] = i\epsilon_{ijk} \mathbf{P}^k, \quad (3)$$

$$[\mathbf{J}^i, H] = 0, \quad (4)$$

$$[\mathbf{J}^i, \mathbf{J}^j] = i\epsilon_{ijk} \mathbf{J}^k, \quad (5)$$

$$[\mathbf{P}^i, \mathbf{K}^j] = -i\delta_{ij} H, \quad (6)$$

$$[H, \mathbf{K}^i] = -i\mathbf{P}^i, \quad (7)$$

$$[\mathbf{J}^i, \mathbf{K}^j] = i\epsilon_{ijk} \mathbf{K}^k, \quad (8)$$

$$[\mathbf{K}^i, \mathbf{K}^j] = -i\epsilon_{ijk} \mathbf{J}^k. \quad (9)$$

It has been pointed out, as early as in Ref. [3], that the algebra induces non trivial constraints on the form of the Hamiltonian of non-relativistic systems where Poincaré invariance is no longer explicit. Indeed, the algebra has been used in the past to constrain the form of the rel-

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ativistic corrections to phenomenological potentials [4]. A derivation of the constraints induced by Poincaré invariance on the form of the potential of a quantum-mechanical two-body system can be found in [5].<sup>2</sup>

In a relativistic field theory the fields are representations of the Poincaré group and the form of the Poincaré generators may be derived from the symmetric energy-momentum tensor [8].<sup>3</sup> For instance, the Poincaré generators of QCD are given by:

$$H = \int d^3x \bar{\psi} (-i\boldsymbol{\gamma} \cdot \mathbf{D} + m) \psi + \frac{\boldsymbol{\Pi}^a{}^2 + \mathbf{B}^a{}^2}{2}, \quad (10)$$

$$\mathbf{P} = \int d^3x \psi^\dagger (-i\mathbf{D}) \psi + \frac{1}{2} [\boldsymbol{\Pi}^a \times, \mathbf{B}^a], \quad (11)$$

$$\mathbf{J} = \int d^3x \psi^\dagger \left( \mathbf{x} \times (-i\mathbf{D}) + \frac{\boldsymbol{\Sigma}}{2} \right) \psi + \frac{1}{2} \mathbf{x} \times [\boldsymbol{\Pi}^a \times, \mathbf{B}^a], \quad (12)$$

$$\mathbf{K} = -t\mathbf{P} + \int d^3x \psi^\dagger \frac{1}{2} \left\{ \mathbf{x}, \bar{\psi} (-i\boldsymbol{\gamma} \cdot \mathbf{D} + m) \psi + \frac{\boldsymbol{\Pi}^a{}^2 + \mathbf{B}^a{}^2}{2} \right\}, \quad (13)$$

where  $\boldsymbol{\Pi}_a^i$  is the canonical variable conjugated to  $\mathbf{A}_{ia}$ . All generators are defined up to a unitary transformation.

In the following I will discuss the realization of the algebra (1)-(9) in the HQET and pNRQCD, summarizing the findings of [9]. In a non-

<sup>2</sup>The system studied in [5] corresponds to pNRQCD in the non-perturbative regime [6,7].

<sup>3</sup>If  $\Theta^{\mu\nu}$  is the symmetric energy-momentum tensor, then

$$\begin{aligned} H &= \int d^3x \Theta^{00}, & \mathbf{P}^i &= \int d^3x \Theta^{i0}, \\ \mathbf{J}^i &= \frac{1}{2} \int d^3x \epsilon_{ijk} (\mathbf{x}^j \Theta^{k0} - \mathbf{x}^k \Theta^{j0}), \\ \mathbf{K}^i &= -t\mathbf{P}^i + \int d^3x \mathbf{x}^i \Theta^{00}. \end{aligned}$$

relativistic EFT, invariance under Lorentz transformations is explicitly broken. The Lorentz-boost generators of the EFT can be constructed by matching order by order with the Lorentz-boost generators of QCD (13). Rotation and space translation are instead preserved symmetries in the EFT, therefore the generators are exactly known and are the generators of QCD projected on the Hilbert space of the EFT. Since the EFT is equivalent to QCD, i.e. a relativistic field theory, the Poincaré algebra generators constructed in this way must still satisfy the Poincaré algebra (1)-(9). This induces non trivial constraints on the form of the interaction. More specifically, it induces some exact relations among the Wilson coefficients of the EFT.

### 3. HQET

We quantize the HQET in the  $A^0 = 0$  gauge. The pairs of canonical variables are  $(\psi, i\psi^\dagger)$ ,  $(\chi, i\chi^\dagger)$  and  $(\mathbf{A}_{ia}, \boldsymbol{\Pi}_a^i)$ . The physical states are constrained by the Gauß law.

The construction of the generators proceeds in the following way. The generators  $H$ ,  $\mathbf{P}$  and  $\mathbf{J}$  can be derived from the symmetric energy-momentum tensor. Since translational and rotational invariance remain exact symmetries when going to the effective theory, the transformation properties of the new particle fields under these symmetries are the same as in the original theory. The derivation of the Lorentz-boost generators is more problematic, since the non-relativistic expansion has destroyed the manifest covariance under boosts. A consistent way to construct  $\mathbf{K}$  is to write down the most general expression, containing all operators consistent with its symmetries and to match it to the QCD Lorentz-boost generators. Accordingly, matching coefficients, typical of  $\mathbf{K}$ , will appear.

We obtain:

$$\begin{aligned} h \equiv \psi^\dagger & \left( m - c_1 \frac{\mathbf{D}^2}{2m} - c_2 \frac{\mathbf{D}^4}{8m^3} - c_F g \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2m} \right. \\ & \left. - c'_D g \frac{[\mathbf{D} \cdot, \boldsymbol{\Pi}]}{8m^2} - i c_S g \frac{\boldsymbol{\sigma} \cdot [\mathbf{D} \times, \boldsymbol{\Pi}]}{8m^2} + \dots \right) \psi \end{aligned}$$

$$+ \frac{\boldsymbol{\Pi}^a \cdot \boldsymbol{\Pi}^a + \mathbf{B}^a \cdot \mathbf{B}^a}{2} - \frac{d'_3}{m^2} f_{abc} g F_{\mu\nu}^a F_{\mu\alpha}^b F_{\nu\alpha}^c \Big|_{\mathbf{E} = \boldsymbol{\Pi}} + \dots,$$

$$H = \int d^3x h, \quad (14)$$

$$\mathbf{P} = \int d^3x \left( \psi^\dagger (-i\mathbf{D}) \psi + \frac{1}{2} [\boldsymbol{\Pi}^a \times, \mathbf{B}^a] \right), \quad (15)$$

$$\mathbf{J} = \int d^3x \left( \psi^\dagger \left( \mathbf{x} \times (-i\mathbf{D}) + \frac{\boldsymbol{\sigma}}{2} \right) \psi + \frac{1}{2} \mathbf{x} \times [\boldsymbol{\Pi}^a \times, \mathbf{B}^a] \right), \quad (16)$$

$$\mathbf{K} = -t\mathbf{P} + \int d^3x \frac{\{\mathbf{x}, h\}}{2} - k^{(1)} \int d^3x \frac{1}{2m} \psi^\dagger \frac{\boldsymbol{\sigma}}{2} \times (-i\mathbf{D}) \psi + \dots \quad (17)$$

The one-loop expressions in the  $\overline{\text{MS}}$  scheme for the coefficients  $c_F$ ,  $c'_D$ ,  $c_S$  and  $d'_3$  can be found in [10] (according to the definitions of  $c'_D$  and  $d'_3$  given in [7]). The coefficient  $k^{(1)}$  is a matching coefficient specific of  $\mathbf{K}$ . In principle,  $k^{(1)}$  may be calculated at any order in perturbation theory by matching (17) to (13). The tree level value can be also calculated by performing a Foldy–Wouthuysen transformation on the Lorentz-boost generators of QCD, in the same way as the tree-level matching coefficients of the HQET Hamiltonian can be derived. At tree level we have  $k^{(1)} = 1$ .

Let us now consider the constraints induced by the Poincaré algebra (1)-(9) on the HQET generators  $H$  and  $\mathbf{K}$ . The constraint  $[\mathbf{P}^i, \mathbf{K}^j] = -i\delta_{ij}H$  has been already used in Eq. (17). Indeed, this commutation relation forces  $\mathbf{K}$  to have the form  $\int d^3x \{ \mathbf{x}, h(\mathbf{x}, t) \} / 2 + \text{translational-invariant terms}$ . From  $[\mathbf{K}^i, \mathbf{K}^j] = -i\epsilon_{ijk}\mathbf{J}^k$  at  $\mathcal{O}(1/m^0)$  it follows that

$$k^{(1)} = 1. \quad (18)$$

From  $[H, \mathbf{K}^i] = -i\mathbf{P}^i$  at  $\mathcal{O}(1/m^0)$  it follows that

$$c_1 = 1, \quad (19)$$

and at  $\mathcal{O}(1/m)$

$$2c_F - c_S - 1 = 0. \quad (20)$$

Finally from  $[H, \mathbf{K}^i] = -i\mathbf{P}^i$  at  $\mathcal{O}(\nabla^2 \nabla^i / m^2)$  we obtain

$$c_2 = 1. \quad (21)$$

All other commutation relations are satisfied at the order we are working. The constraints (19), (20) and (21) were first derived in the framework of reparametrization invariance in [2,10].

#### 4. pNRQCD

The pNRQCD Lagrangian for a heavy quark-antiquark system is obtained from the NRQCD Lagrangian [11] by integrating out the soft degrees of freedom associated with the scale of the relative momentum of the two heavy quarks in the bound state [12]. The name pNRQCD has been used in the literature to identify effective field theories with different degrees of freedom. Here we call pNRQCD the effective field theory that can be obtained from NRQCD by perturbative matching and contains, as degrees of freedom, the quark-antiquark field (that can be split into a colour singlet  $S = S\mathbf{1}_c / \sqrt{N_c}$  and a colour octet  $O = O^a \mathbf{T}^a / \sqrt{T_F}$  component) and (ultrasoft) gluons. The fields  $S$  and  $O^a$  are functions of  $(\mathbf{X}, t)$  and  $\mathbf{x}$ , where  $\mathbf{X}$  is the centre-of-mass coordinate and  $\mathbf{x}$  the relative coordinate. The coordinate  $\mathbf{x}$  plays the role of a continuous parameter, which specifies different fields. All the gauge fields have been multipole expanded around the centre-of-mass. Therefore, the terms in the pNRQCD Lagrangian are organized in powers of  $1/m$  and  $x$ .

The canonical variables and their conjugates are  $(S, iS^\dagger)$ ,  $(O_a, iO_a^\dagger)$ , and  $(\mathbf{A}_{ia}, \boldsymbol{\Pi}_a^i)$ . The physical states are constrained to satisfy the Gauß law.

As in the case of the HQET, since translational and rotational invariance are exact symmetries of the effective theory, the generators  $H$ ,  $\mathbf{P}$  and  $\mathbf{J}$  can be derived from the symmetric energy-momentum tensor. The pNRQCD Lorentz-boost generators  $\mathbf{K}$  can be derived by writing down the most general expressions containing all operators consistent with their symmetries and by matching them to the NRQCD Lorentz-boost generators, which, at the order we are working here, are the simple extension of Eq. (17) to the case of one particle and one antiparticle.

We obtain:

$$h \equiv \frac{\Pi^{a2} + \mathbf{B}^{a2}}{2} + \int d^3x \text{Tr} \left\{ S^\dagger (2m + h_S) S \right. \\ + O^\dagger (2m + h_O) O \\ + [(S^\dagger h_{SO} O + \text{H.C.}) + \text{C.C.}] \\ + [O^\dagger h_{OO} O + \text{C.C.}] \\ \left. + [O^\dagger h_{OO}^A O h_{OO}^B O + \text{C.C.}] \right\},$$

$$H = \int d^3X h, \quad (22)$$

$$\mathbf{P} = \int d^3X \int d^3x \text{Tr} \left\{ S^\dagger (-i \nabla_X) S \right. \\ + O^\dagger (-i \mathbf{D}_X) O \left. \right\} \\ + \frac{1}{2} \int d^3X [\Pi^a \times, \mathbf{B}^a], \quad (23)$$

$$\mathbf{J} = \int d^3X \int d^3x \text{Tr} \left\{ S^\dagger \left( \mathbf{X} \times (-i \nabla_X) \right. \right. \\ + \mathbf{x} \times (-i \nabla_x) + \frac{\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}}{2} \left. \right) S \\ + O^\dagger \left( \mathbf{X} \times (-i \mathbf{D}_X) \right. \\ + \mathbf{x} \times (-i \nabla_x) + \frac{\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}}{2} \left. \right) O \left. \right\} \\ + \frac{1}{2} \int d^3X \mathbf{X} \times [\Pi^a \times, \mathbf{B}^a], \quad (24)$$

$$\mathbf{K} = -t \mathbf{P} + \int d^3X \frac{1}{2} \{ \mathbf{X}, h \} \\ + \int d^3X \int d^3x \text{Tr} \left\{ [S^\dagger \mathbf{k}_{SS} S + \text{C.C.}] \right. \\ + [(S^\dagger \mathbf{k}_{SO} O + \text{H.C.}) + \text{C.C.}] \\ \left. + [O^\dagger \mathbf{k}_{OO} O + \text{C.C.}] \right\}, \quad (25)$$

where C.C. stands for charge conjugation and H.C. for Hermitian conjugation. Explicit expressions for  $h_S$ ,  $h_O$ ,  $h_{SO}$ ,  $h_{OO}$  and  $h_{OO}^{A,B}$  at order  $x^2/m^0$ ,  $x^0/m$ ,  $(x/m) \nabla_X$  and  $(x^0/m^2) \nabla_X$  and for  $\mathbf{k}_{SS}$ ,  $\mathbf{k}_{SO}$  and  $\mathbf{k}_{OO}$  at order  $x^2/m^0$ ,  $x^0/m$  and  $(x/m) \nabla_X$  can be found in [9].

Let us consider the constraints induced by the Poincaré algebra (1)-(9) on the pNRQCD generators  $H$  and  $\mathbf{K}$ . The constraint  $[\mathbf{P}^i, \mathbf{K}^j] = -i \delta_{ij} H$  has been already used in writing Eq. (25). Indeed, it forces  $\mathbf{K}$  to have the form  $\int d^3X \{ \mathbf{X}, h(\mathbf{X}, t) \} / 2 +$  translational-invariant terms. From the constraints that we get on  $\mathbf{K}$  from the other commutation relations and using the freedom that we have to redefine the Poincaré generators via a unitary transformation we obtain at the order we are working:

$$\mathbf{k}_{SS} = -\frac{1}{4m} \left( \boldsymbol{\sigma}^{(1)} \times (-i \nabla_x) \right) \\ - \frac{1}{8m} \{ \mathbf{x}, \nabla_X \cdot \nabla_x \}, \quad (26)$$

$$\mathbf{k}_{OO} = -\frac{1}{4m} \left( \boldsymbol{\sigma}^{(1)} \times (-i \nabla_x) \right) \\ - \frac{1}{8m} \{ \mathbf{x}, \mathbf{D}_X \cdot \nabla_x \} \\ - \frac{1}{8} k_{OOa}^{(0,2)}(x) \mathbf{x} (\mathbf{x} \cdot g \Pi) \\ - \frac{1}{8} k_{OOb}^{(0,2)}(x) \mathbf{x}^2 g \Pi, \quad (27)$$

$$\mathbf{k}_{SO} = 0, \quad (28)$$

where  $k_{OOa}^{(0,2)}$  and  $k_{OOb}^{(0,2)}$  are some matching coefficients specific of  $\mathbf{K}$ . At tree level we have  $k_{OOa}^{(0,2)} = 1$  and  $k_{OOb}^{(0,2)} = 0$ . The Poincaré algebra also constrains the form of the pNRQCD Lagrangian. In the following, I will discuss the different type of constraints.

**(A) Kinetic energy.** The centre-of-mass kinetic energy is fixed to be equal to  $-\nabla_X^2/4m$ . We note that Poincaré invariance by itself does not constrain the coefficient of the kinetic energy of the quarks in the centre-of-mass frame.

**(B) Potentials of order  $1/m^2$ .** If we call  $V^{(0)}$  the static potential,  $V_{\mathbf{p}^2}$  the  $1/m^2$  momentum square dependent potential,  $V_{\mathbf{L}^2}$  the  $1/m^2$  angular momentum square potential and  $V_{LS}$  the  $1/m^2$  spin-orbit potential either in the singlet ( $S$ ) or in the octet ( $O$ ) sector we obtain:

$$\frac{V_{LS Sa}}{V_S^{(0)'}} = \frac{V_{LS Oa}}{V_O^{(0)'}} = 1, \quad (29)$$

$$V_{\mathbf{L}^2 Sa} + \frac{x V_S^{(0)'}}{2} = V_{\mathbf{L}^2 Oa} + \frac{x V_O^{(0)'}}{2} = 0, \quad (30)$$

$$V_{\mathbf{p}^2 Sa} + V_{\mathbf{L}^2 Sa} + \frac{V_S^{(0)}}{2} =$$

$$V_{\mathbf{p}^2 Oa} + V_{\mathbf{L}^2 Sa} + \frac{V_O^{(0)}}{2} = 0, \quad (31)$$

where the label  $a$  is kept for consistency with the notation of [9]. Eq. (29) in the singlet sector is the relation between the spin-orbit potentials and the static potential first derived in [13]. Eqs. (30)-(31) in the singlet sector are the relations between the momentum-dependent potentials first derived in [14]. They were also obtained in [5]. A lattice check of these relations has been done in [15]. The extension to the octet sector has been derived in [9].

### (C) Singlet and octet couplings to gluons.

In the sector of the pNRQCD Lagrangian containing the couplings of the heavy-quarkonium fields to the gluons we may distinguish two set of relations induced by the Poincaré invariance. The first set constrains the chromoelectric field to enter the Lagrangian just in the combination

$$\mathbf{x} \cdot \left( g\mathbf{E} + \frac{1}{2} \left\{ \frac{-i\mathbf{D}_X}{2m} \times, g\mathbf{B} \right\} \right), \quad (32)$$

i.e. like in the Lorentz force. The second set contains relations that involve combinations of matching coefficients appearing at different orders in the expansion in  $1/m$  and  $x$ :

$$\frac{2c_F V_{SO}^{(1,0)} - c_s V_{SO}^{(2,0)}}{V_{SO}^{(0,1)}} = 2 \frac{c_F V_{OO}^{(1,0)} + V_{O\otimes O}^{(1,0)}}{V_{OO}^{(0,1)}} - \frac{c_s V_{OO}^{(2,0)} + V_{O\otimes O}^{(2,0)}}{V_{OO}^{(0,1)}} = 1, \quad (33)$$

$$2V_{SO}^{(1,0)} - V_{SO}^{(2,0)} = 2 \left( V_{OO}^{(1,0)} + V_{O\otimes O}^{(1,0)} \right) - \left( V_{OO}^{(2,0)} + V_{O\otimes O}^{(2,0)} \right) = 0, \quad (34)$$

$$\frac{-V_{SO}^{(2,0)}}{x V_{SO}^{(0,1)'}} = \frac{-V_{OO}^{(2,0)} - V_{O\otimes O}^{(2,0)'}}{x V_{OO}^{(0,1)'}} = 1, \quad (35)$$

$$V_{SO}^{(2,0)} = V_{OO}^{(2,0)} + V_{O\otimes O}^{(2,0)} = 0, \quad (36)$$

$$V_{OO}^{(1,0)} = 1 + \frac{V_{OO}^{(2,0)} - V_{O\otimes O}^{(2,0)'}}{2}, \quad (37)$$

where  $V^{(i,j)}$  are the matching coefficients that appear at order  $x^j/m^i$ , the letters  $a, b, \dots$  label different operators explicitly listed in [9] and

the specifications  $SO$  and  $OO$  refer to couplings with gluons in the singlet-octet and octet-octet sector respectively. Eq. (33) involves combinations of matching coefficients inherited from HQET/NRQCD. Somehow this relation reflects at the level of the pNRQCD matching coefficients the relation (20) among the HQET matching coefficients. Eq. (35) involves derivatives of  $V_{SO}^{(0,1)}$  and  $V_{OO}^{(0,1)}$ . Eq. (37) is typical for the non-Abelian structure of QCD.

## 5. OUTLOOK

In this contribution I have addressed two questions: what is the relation between reparametrization invariance and Poincaré invariance in the HQET; may Poincaré invariance be used to constrain the form of other non-relativistic effective field theories.

For what concerns the first question, in Sec. 3 we have shown that by imposing the Poincaré algebra on the generators of the Poincaré transformations of the HQET we obtain the relations, Eqs. (18)-(21), derived formerly from reparametrization invariance. This shows that reparametrization invariance is, indeed, one way in which the Poincaré invariance of QCD manifests itself in the HQET (see also [16]).

The second question may be of particular interest when very little is known about the relation between the effective degrees of freedom and the original ones. This is, for instance, the case when dealing with EFTs in the non-perturbative domain. In Sec. 4 we have outlined the calculation of the constraints induced by the Poincaré invariance on the form of the pNRQCD Lagrangian. Some of the obtained relations in the singlet potential sector of the pNRQCD Lagrangian were already derived in the literature by explicitly boosting their expression in terms of Wilson loop operators. The same relations also apply to the octet sector. New relations involving the couplings of the singlet and octet fields with the gluons were derived in [9] and are displayed in Eqs. (32)-(37).

Another, quite natural, non-relativistic EFT where to apply the above method would be NRQCD. Up to order  $1/m$  the NRQCD La-

grangian in the particle and antiparticle sector coincide with that one of the HQET and, therefore, the relations induced by Poincaré invariance are the same. Starting from the order  $1/m^2$ , the NRQCD Lagrangian contains four-fermion operators, which are responsible for decay and production processes. Poincaré invariance will constrain the form of these operators. In general, the presented approach, may be suited to derive exact relations among the matching coefficients of all effective field theories, where the manifest covariance under boosts has been destroyed by an expansion in some small momenta, like the soft-collinear effective theory [17].

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